

**Collaborative MA Programme in Economics for Anglophone Africa
(Except Nigeria)**

JOINT FACILITY FOR ELECTIVES

JUNE – SEPTEMBER 2007

ECONOMETRICS THEORY & PRACTICE II

Second Semester: Final Examination

Time: 09.00 AM – 12.00 Noon

Date: Wednesday, September 26, 2007

INSTRUCTION:

Answer any FOUR questions

Question 1

Given the linear regression model

$$Y = X\beta + \mu$$

where Y is $n \times 1$ vector, X is $n \times k$ matrix, β is $k \times 1$ vector, and μ is $n \times 1$ vector containing the error terms.

- 1.1** Use the maximum likelihood (ML) method to find estimators for the unknown parameter vector θ (β s and σ^2). **(4 points)**
- 1.2** Derive the variance of the maximum likelihood estimator $\hat{\theta}_{ML}$. **(4 points)**
- 1.3** Use the generalised method of moments (GMM) to estimate the vector $\hat{\beta}$. **(4 points)**
- 1.4** Show that the GMM estimator for the variance σ^2 is consistent. **(3 points)**

Question 2

Zehra Kasanakoglu (1978) used the following recursive model to analyse the determinants of male earnings differential in Turkey¹

$$\begin{aligned}E &= \alpha_1 + \alpha_2 L + \alpha_3 F + \alpha_4 R + \mu_1 \\O &= \beta_1 + \beta_2 E + \beta_3 F + \beta_4 R + \mu_2 \\Y &= \delta_1 + \delta_2 E + \delta_3 O + \delta_4 F + \delta_5 A + \delta_6 U + \mu_3\end{aligned}$$

where E is years of schooling, O is occupation index, Y is the annual income, L is father's literacy, F is father's occupation index, R is region of origin, A is age, U is place of current residence, and μ is the error term.

- 2.1 Check the identifiability of model equations using the order and the rank conditions. **(5 points)**
- 2.2 Which of the parameters in the model are consistently estimable, using what estimation method, and under what assumptions? Explain. **(5 points)**
- 2.3 The recursive model above is estimated using OLS. The output below gives results of regressing estimated errors of one equation on errors from the other two equations. Guided by this output show which estimated coefficients in the recursive model can possibly be inconsistent (estimated coefficients of the recursive model are not reported here). Suggest a method (s) of estimation that yields consistent estimators.

$$e_1 = 1.035 + 0.074 * e_2 - 0.000006 * e_3$$

(-10.4) (19.13) (-0.67)

$$R^2 = 0.2336$$

$$e_2 = 13.95 + 3.14 * e_1 + 0.000014 * e_3$$

(25.71) (19.13) (0.31)

$$R^2 = 0.2333$$

$$e_3 = 300.057 - 59.594 * e_1 + 4.319 * e_2$$

(0.94) (-0.67) (0.31)

$$R^2 = 0.0004$$

$$\rho_{13} = -0.017 \quad \rho_{12} = 0.483 \quad \rho_{23} = 0.0001$$

- Values in parentheses are coefficient respective t-values. ρ denotes simple correlation coefficients. e_1 , e_2 , and e_3 are estimated errors from the schooling, occupation and income equations, respectively. **(5 points)**

¹ A simultaneous model approach to the determinants of male earnings differentials in Turkey, the Review of Economics and Statistics, vol. 60, no. 2. pp. 307-312.

Question 3

A simple consumption model is estimated for Kenya over the period 1967-2003, using annual data. Consumption is assumed to be forward looking and linearly related to permanent or expected future income

$$C_t = \beta_1 + \beta_2 Y_t^* + \varepsilon_t$$

where C is real private consumption, Y is real income (measured by GDP), and ε is the error term and is assumed to follow the classical assumptions. Since Y^* is not observable, expected or permanent income is assumed to be generated by the following adaptive expectations mechanism

$$Y_t^* - Y_{t-1}^* = \gamma(Y_t - Y_{t-1}^*)$$

where γ is the adjustment coefficient ($0 < \gamma \leq 1$). Using the adjustment mechanism the estimable regression is

$$C_t = \alpha_1 + \alpha_2 Y_t + \alpha_3 C_{t-1} + v_t.$$

The estimated model and estimation results (using OLS) are given below

$$C_t = 422.4 + 0.4228 * Y_t + 0.4937 * C_{t-1}$$

(1.6) (5.09) (5.02)

$$R^2 = 0.942, \quad F = 275.3, \quad DW = 1.42$$

t-values in parentheses. Number of observations is 37.

- 3.1 Show how the estimable model is derived using the adaptive expectations adjustment rule. **(3 points)**
- 3.2 Determine or compute the short-run and the long-run marginal propensities to consume (MPC) and show the conceptual difference between them using given results. Comment on the speed of adjustment. **(4 points)**
- 3.3 Are OLS estimators consistent in this adaptive expectations model? Explain. Show how you would test for autocorrelation in v_t . **(4 points)**
- 3.4 Assume that the following ARDL (1,1) model is used to model consumption expenditure

$$C_t = \alpha_2 Y_t + \alpha_3 Y_{t-1} + \alpha_4 C_{t-1} + v_t.$$

Show how to test for the presence of common factor in the above model? What are the implications of the presence of common factor (if found to be present in consumption and income polynomials) for the consumption specification. **(4 points)**

Question 4

4.1 Random walk process is a special case of processes with stochastic trend. True or False? Explain. **(2 points)**

4.2 Using Yule-Walker equations derive the autocorrelation function $\rho(k)$ for the autoregressive process

$$Y_t = 1.0*Y_{t-1} - 0.8*Y_{t-2} + \varepsilon_t$$

for $k = 0, 1, 2, 3$.

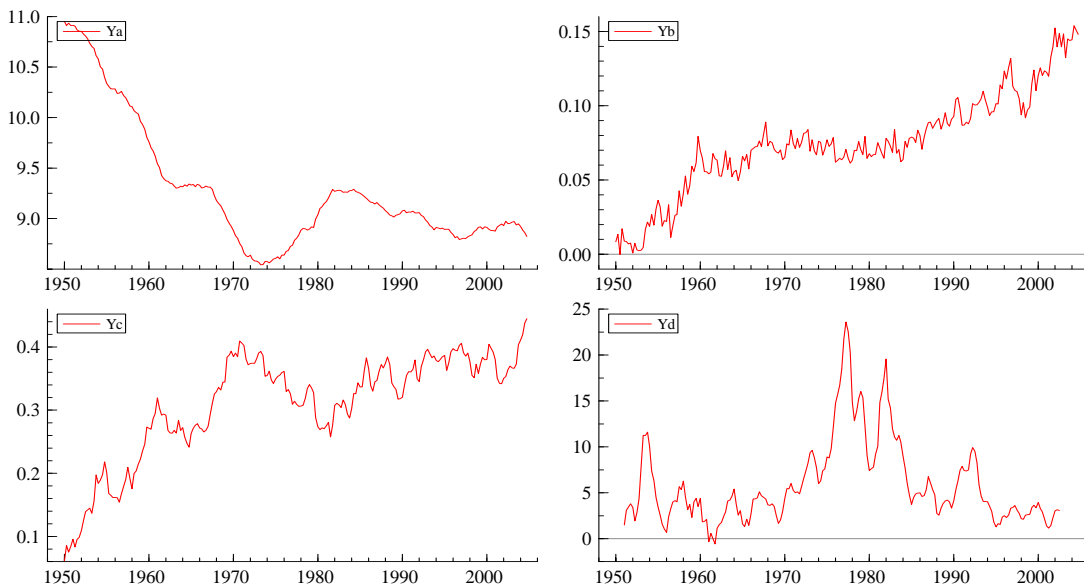
(3 points)

4.3 Derive the variance and covariance for the AR (1) below and comment on stationarity requirements for the process

$$Y_t = \alpha Y_{t-1} + \varepsilon_t.$$

(5 points)

4.4 Comment on the stochastic properties of time profiles of series Y_a , Y_b , Y_c , and Y_d shown below and write a linear stochastic equation that best describes the behaviour of each of the series.



(5 points)

Question 5

- 5.1 Show how to test for cointegration in the single equation model below using Engle-Granger two-stage method. Write the error correction representation for the given model and explain the economic meaning of the correction process.

$$Y_t = \alpha + \beta X_t + \varepsilon_t. \quad (4 \text{ points})$$

- 5.2 Use a two-equation-two-lags vector autoregressive (VAR) system to discuss the implications of the orthogonalisation process for analysing policy scenarios. Then use a two-equation-one-lag system to derive the impulse response functions. (4 points)

- 5.3 Estimation results for two variable-one-lag VAR model $\mathbf{Z}_t = \mathbf{A}\mathbf{Z}_{t-1} + \boldsymbol{\varepsilon}_t$ are given below

$$\begin{aligned} Y_{1,t} &= 1.2 * Y_{1,t-1} - 0.2 * Y_{2,t-1} + \varepsilon_{1,t} \\ Y_{2,t} &= 0.6 * Y_{1,t-1} + 0.4 * Y_{2,t-1} + \varepsilon_{2,t} \end{aligned}$$

Find the eigenvalues λ_i and respective eigenvectors. What can you infer from the results about cointegration in the system? (4 points)

- 5.4 Show how Dickey-Fuller and Augmented Dickey-Fuller equations are applied to test for nonstationarity of time series. (3 points)